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Research Article

Separation of Regional Magnetic Anomaly using a Isoparametric Element with Third-order Twelve-node

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Abstract

Magnetic prospecting is one of the most widely used geophysical prospecting methods in the exploration of fuel resources including crude oil and coal. In magnetic prospecting data process, anomaly separation is an important step. A method for computation of regional magnetic anomaly (RMA) using a third-order isoparametric element with 12 nodes is developed. In this technique, regional and residual magnetic anomaly can be separated fully using given magnetic data even in the case of irregular-shaped survey area.

Key words: magnetic prospecting; anomaly separation;regional anomaly; third-order isoparametric element

1. Introduction

In magnetic prospecting data process, anomaly separation is an important step to improve the accuracy and efficiency of the interpretation. Because the gravity or magnetic field at any observation point records the sum of contributions from all subsurface sources. These anomalies include regional anomaly caused by deeper structures with a relatively large distribution and residual anomaly produced by relatively small target body. Therefore, the regional/residual separation from potential field data is an essential procedure to improve the efficiency and accuracy of gravity and magnetic surveying. Regional-residual separation has been studied for many years and recently representative methods are polynomial fitting, spectral and inversion approaches.

The polynomial fitting is a method of approximating the regional anomaly by a polynomial. Yan Feng et al. (2016) proposed a method modeling the geomagnetic field in a polynomial fitting method and considered the variation of geomagnetic field in China using third-order Tayler polynomial(3DTP) and Surface spline (SS) model. The polynomial fitting method is not suitable because it is difficult to describe the regional anomaly in polynomial terms in case of large survey area and complex geological conditions (Yan et al., 2016).

Spectral methods are the separation of long-wavelength RMA and short-wavelength residual magnetic anomaly through digital filtering in the frequency domain, which can cause spurious RMA depending on the wavelength (Mandal et al., 2018).

Kim Gang-Sop et al. (2018) studied a three-dimensional inversion method (body-growth method) of magnetic anomaly and it has the advantage that anomaly separation can also be performed simultaneously. However, the body-growth (BG) method only determines the first-order polynomial type of regional anomaly, so that the separation of regional anomaly is not possible in complex geological conditions (Kim et al., 2014).

Mallick K (1999) simulated the survey area outline with an isoparametric element and proposed a method to estimate the regional anomaly inside the region by analytical function using only nodal measurements on it (Mallick et al., 1999). But only regional gravity anomaly separation was considered and only a second-order rectangle isoparametric element was used in their work. The interactive FORTRAN program FEAODD.FOR for wide application of finite element method (FEM) to calculate the regional gravity anomaly containing only eight nodes around a rectangular region is introduced (Agarwal et al., 2010).

In this paper, we develop a new method for calculating RMA using a third-order isoparametric element with 12 nodes. The RMA is approximated by the weighted sum of the magnetic field measurements at 12 nodes of a third-order isoparametric element superposed in the survey area. The calculation of RMA is first performed in reference space with the shape function used as a weight function and then converted to a real survey area. In this technique, regional and residual magnetic anomaly can be calculated fully using given magnetic data even in the case of irregular-shaped survey area, and the magnetic field data inside the survey area do not participate in the calculation of regional anomaly.

2. Methodology

An element using the same shape function for field calculation and coordinate transformation is called an isoparametric element.

2.1. Third-order isoparametric element

The field variable of the third-order isoparametric element are expressed by the following equation:

$$f(\xi,\eta)= N_1+N_2\xi+N_3\eta+N_4\xi^2+N_5\xi\eta+N_6\eta^2+N_7\eta^3+N_8\xi^2\eta+N_9\xi\eta^2+N_{10}\eta^3+N_{11}\xi^3\eta+N_{12}\xi\eta^3 \quad (1)$$

where ξ, η are local coordinates and the definition of the element shape function is as follows:

Hence, the shape function of each node is written as

$$\begin{aligned}
&N_1=\frac{1}{32}(1-\xi)(1-\eta)\left(-10+9\left(\xi^2+\eta^2\right)\right), && N_2=\frac{1}{32}(1+\xi)(1-\eta)\left(-10+9\left(\xi^2+\eta^2\right)\right),\\
&N_3=\frac{1}{32}(1-\xi)(1+\eta)\left(-10+9\left(\xi^2+\eta^2\right)\right), && N_4=\frac{1}{32}(1+\xi)(1+\eta)\left(-10+9\left(\xi^2+\eta^2\right)\right),\\
&N_5(\xi,\eta)=\frac{9}{32}\left(1-\xi^2\right)(1-\eta)(1-3\xi), && N_6(\xi,\eta)=\frac{9}{32}\left(1-\xi^2\right)(1-\eta)(1+3\xi),\\
&N_7(\xi,\eta)=\frac{9}{32}\left(1-\xi^2\right)(1+\eta)(1-3\xi), && N_8(\xi,\eta)=\frac{9}{32}\left(1-\xi^2\right)(1+\eta)(1+3\xi),\\
&N_9(\xi,\eta)=\frac{9}{32}\left(1-\eta^2\right)(1-\xi)(1-3\eta), && N_{10}(\xi,\eta)=\frac{9}{32}\left(1-\eta^2\right)(1+\xi)(1-3\eta)\\
&N_{11}(\xi,\eta)=\frac{9}{32}\left(1-\eta^2\right)(1-\xi)(1+3\eta), && N_{12}(\xi,\eta)=\frac{9}{32}\left(1-\eta^2\right)(1+\xi)(1+3\eta)
\end{aligned} \quad (2)$$

2.2. Separation method

In practice, the survey area is irregular due to various conditions. In previous works, they selected a rectangular region inside an irregular-shaped survey area, calculated the regional anomaly inside it or interpolated the measurements to obtain data in a rectangular region, and then calculated the regional anomaly using a rectangular isoparametric element.

We establish a method to approximate the irregular-shaped survey area by a quadrangle and then calculate the RMA using a third-order isoparametric element.

Separation steps are as follows;

- Select four vertices of the quadrangle superposed to the irregular-shaped survey area and transform to a rectangle of the ξ-η reference space by the shape function (for details, we will discuss later.),
- Select 12 nodes representing the RMA on the side of the rectangle as shown in Fig.1,
- Transform the ξ-η coordinates of the selected nodes into the real x-z coordinates,

  Choose the field values of the nearest observation points from transformed 12 nodes as the nodal function values.
- The rectangular region is divided into appropriate intervals and the shape function is calculated according to Eq.3 at each grid point,
- According to Eq.1, calculate the RMA at the grid points,

Here we will discuss in detail the coordinate transformation of a quadrangle to a rectangle.

In order to transform a quadrangle 1234 into a rectangle in the ξ-η reference space, the ξ, η coordinates of each vertex must be as follows:

ξ_m=-1, 1, -1, 1, η_m=-1, -1, 1, 1, m=1, 2, 3, 4 (3)

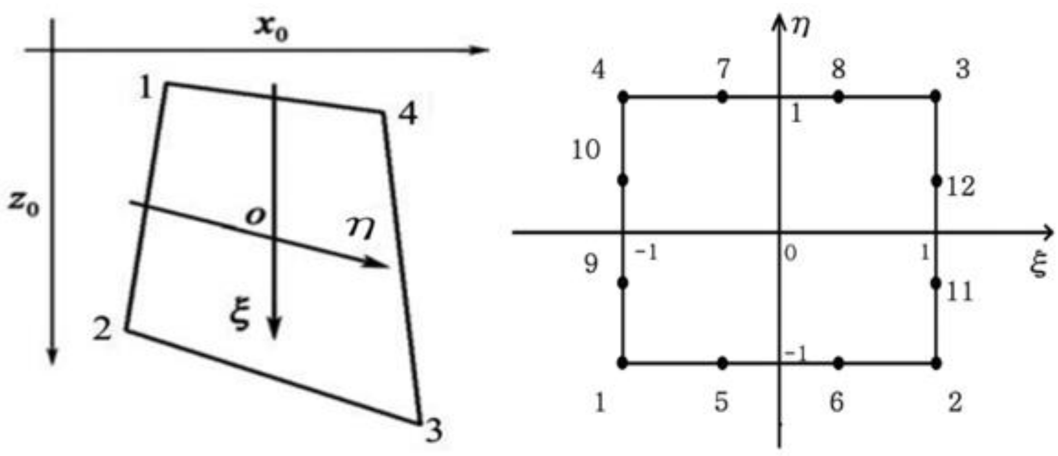

Figure 1: Quadrangle and third-order rectangle isoparametric finite element in reference ξ-η space

In this case x, z are all functions of ξ, η, so they are expressed as:

$$\begin{cases} x=a_1+a_2\xi+a_3\eta+a_4\xi\eta \\ z=b_1+b_2\xi+b_3\eta+b_4\xi\eta \end{cases} \quad (4)$$

Substituting the coordinates of the four vertices into this equation and solving the system of equations

$$\begin{cases} x=\frac{1}{4}\left[(1-\xi)(1-\eta)x_1+(1+\xi)(1-\eta)x_2+(1-\xi)(1+\eta)x_3+(1+\xi)(1+\eta)x_4\right] \\ z=\frac{1}{4}\left[(1-\xi)(1-\eta)z_1+(1+\xi)(1-\eta)z_2+(1-\xi)(1+\eta)z_3+(1+\xi)(1+\eta)z_4\right] \end{cases} \quad (5)$$

Here, the coefficients of x_m or z_m are as follows;

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta), N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1-\xi)(1+\eta), N_4 = \frac{1}{4}(1+\xi)(1+\eta) \quad (6)$$

From Eq.5, we find the real coordinates of the interpolation nodes on four sides of a rectangle in the ξ-η reference space, and then the measured values at that point are the field values of the interpolation nodes. Then, according to Eq. 1, the RMA is calculated and subtracted from the measured field to obtain the residual magnetic anomaly.

We proceed all processes such as interpolation node selection, coordinate transformation, selection of the nearest measurement point and field value selection in the ξ-η reference space from the selection of four quadrilateral vertices in Matlab2021a environment.

This method estimates RMA only with points on the outline of the survey area, thus reducing RMA distortion compared to all other previous methods participated all points of the region, and there is no need for regional and residual structural assumptions. In addition, this method has the advantage of constructing an isoparametric element and selecting nodes, where the RMA is determined by weighted sum of the nodal function values.

3. Application to field magnetic data

A new method for separation of RMA based on a third-order isoparametric element was applied to a magnetite deposit in Yonsan County, North HwangHae Province.

The host rock for the deposit is the Pirangdong Formation (phyllite, quartzite, limestone) in the Yontan Group of the Neoproterozoic era, and the Neoproterozoic Yonsan intrusive (gabbro-diabase, diabase and gabbro diorite) is widely distributed in the northern part of the survey area. The main fault is the Yonsan and Hachol faults extending east-west, and the ore body is closely related to the Hachol fault. The main ore body is a typical mafic quartzite magnetite entering the Pirangdong Formation, and the iron ore deposit is a sedimentary metamorphic deposit.

Magnetic data at 888 stations are measured in survey area at 10 m spacing along 18-profiles nearly oriented in north–south direction and spaced nearly 20-100 m apart from each other.

The topographic relief of survey area and observed magnetic contour map is shown in Figure. 2.

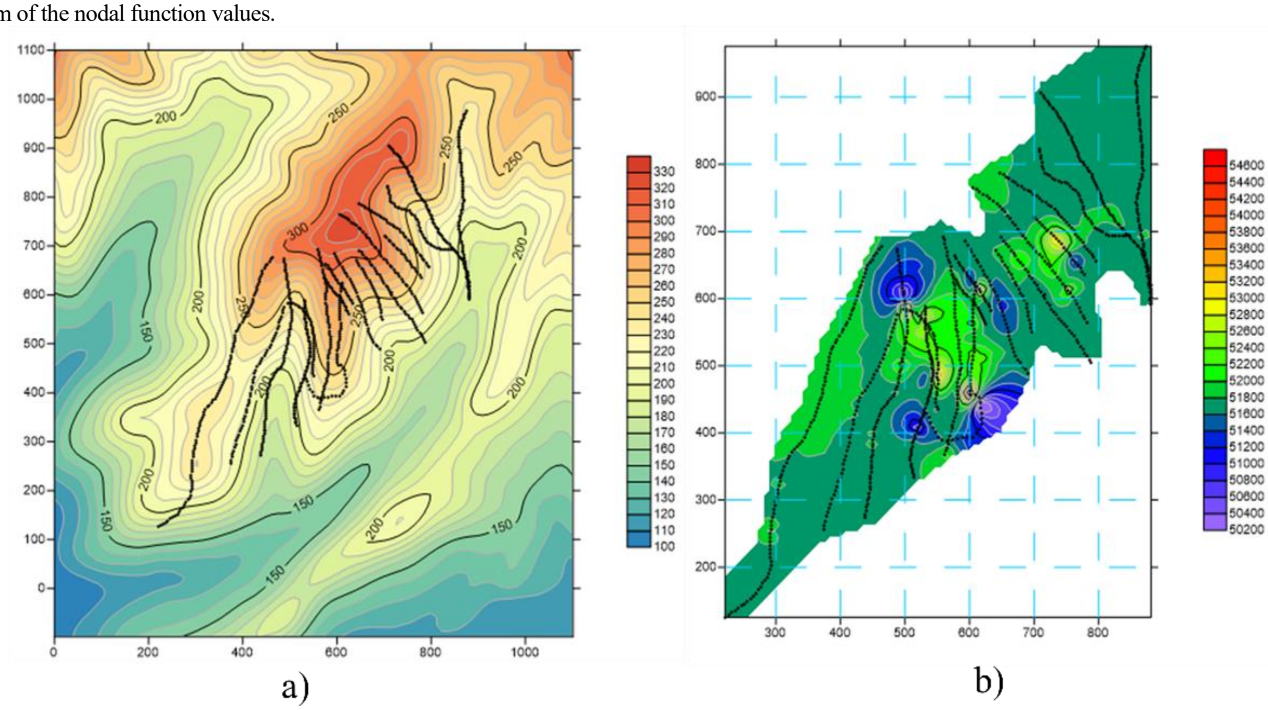

Figure 2: Topographic relief of survey area and observed magnetic field contour map

As shown in Figure.2, the layout of 888 stations is not sufficient to apply the rectangular isoparametric element. To apply the new method proposed in this paper, we first select a quadrangular region superposed to the survey area and obtain its four vertices, and then transform the coordinates by a shape function into a rectangle in the reference space. Then, for a rectangle, we compute the shape function at 12 nodes. Then, from the given stations, a third-order isoparametric element is formed by taking the nearest stations from the selected nodes 1-12.

The regional and residual magnetic anomaly contours calculated from the observations nearest to the 12 nodes of a third-order isoparametric element are shown in Figure. 3. As shown in this figure, the RMA tend to gradually weaken from west to east, and the residual anomaly appear concentrated in the central and eastern orebody regions.

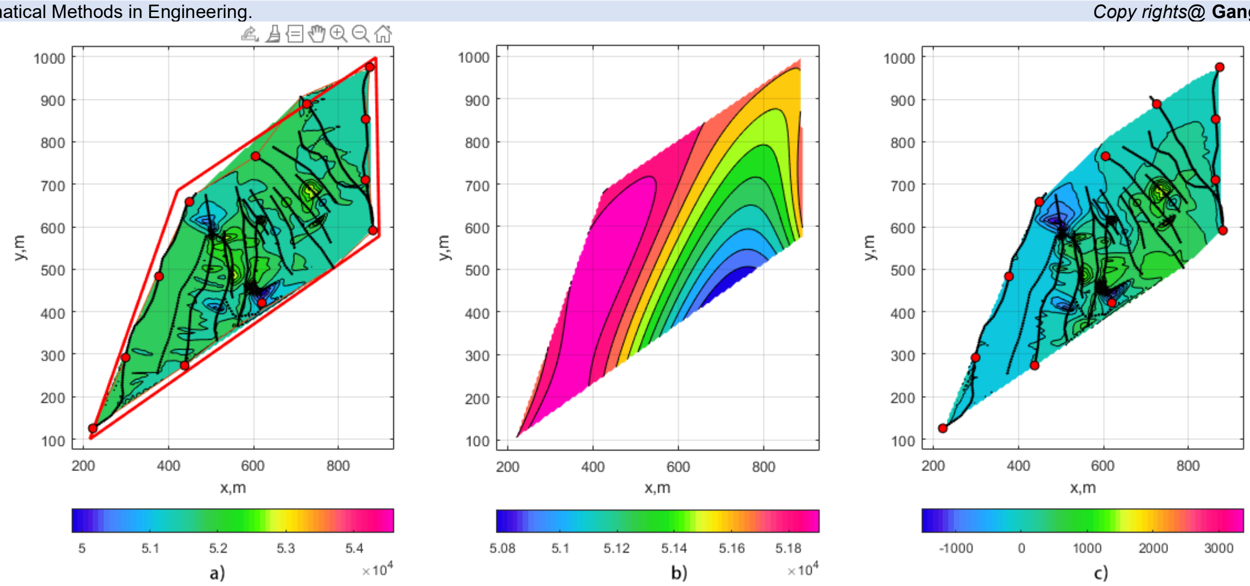

Figure 3: Observed magnetic field contour map and quadrangle (red line), interpolation nodes (red circle) (a), RMA(b) and residual anomaly contour map(c)

4. Conclusion

Once the element is selected, the regional anomaly is determined completely by a third-order analytic function from its element node values, and it is not only possible to reduce residual anomaly distortion without the need for regional and residual structure assumptions, but also has the advantage of being applicable to irregular-shaped survey area and any data layout.

In this method, it is most important to select the finite element node so that it can fully reflect the regional anomaly characteristics inside the survey area. This method can be used to calculate RMA as well as regional gravity anomaly.

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