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Research Article

Analysis and Control of Cigarette Smoking and Alcoholism Models

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Abstract

Cigarettes and alcohol are detrimental to human health and are among the leading causes of death today. Both cigarette smoking and alcoholism are addictive and need to be understood and controlled effectively. This paper presents a mathematical framework involving bifurcation analysis and multiobjective nonlinear model predictive control for two models, the first involving cigarette smoking and the second involving alcoholism. Bifurcation analysis is a powerful mathematical tool used to address the nonlinear dynamics of any process. Several factors must be taken into account, and multiple objectives must be achieved simultaneously. The MATLAB program MATCONT was utilized to conduct the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the advanced global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the presence of limit and branch points in the two models. These limit and branch points are advantageous as they allow the multiobjective nonlinear model predictive control calculations to converge to the Utopia point, which represents the most beneficial solution. The combination of bifurcation analysis and multiobjective nonlinear model predictive control for models involving cigarette smoking and alcoholism is the main contribution of this paper.

Key Words: alcoholism; cigarette smoking; bifurcation; optimization; control

Background

Lahrouz et al (2011) [1] discussed the Deterministic and Stochastic Stability of a Mathematical Model of Smoking. Bhunu (2012) [2] developed a mathematical analysis of alcoholism. Mulonea and Straughan (2012) [3] developed and tested a model about binge drinking. Wang et al (2016) [4] provided optimal control strategies in an alcoholism model. Ullah et al (2016) [5] discussed the dynamical Features of a Mathematical Model on Smoking. Sikander et al. (2017) [6] produced Optimal Solutions for a bio-mathematical model of the Evolution of smoking habits. Ur Rahman et al (2018) [7], discussed the threshold dynamics and optimal control of an age-structured model for stopping smoking. Mu'tamar Khozin(2018) [8] developed an optimal control strategy for the alcoholism model with two infected compartments**.** Uçar et al (2018) [9] conducted a mathematical analysis and numerical simulation for a smoking model with Atangana-Baleanu derivative. Sun and Jiav (2019) [10], discussed the optimal control of a delayed smoking model with immigration. Mahdy et al (2020) [11] studied the dynamical characteristics and signal flow graph of nonlinear fractional smoking mathematical model. Zhang et al (2020) [12] studied the harmonic mean type dynamics of a delayed giving up smoking model and optimal control strategy via legislation. Mahdy et al (2020) [13] developed an approximate solution for solving nonlinear fractional order smoking model. Ilmayasinta and Purnawan, H. (2021) [14] performed Optimal Control in a Mathematical Model of Smoking.

This paper aims to perform bifurcation analysis in conjunction with multiobjective nonlinear model predictive control (MNLMPC) for the smoker model (Ilmayasinta and Purnawan, H. (2021) [14]) and the alcoholism model with two infected compartments Mu'tamar Khozin(2018) [8]. This paper is organized as follows. First, the model equations are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) are then described. This is followed by the results and discussion, and conclusions.

1. Model Equations

In this section, details of the smoker model (Ilmayasinta and Purnawan, H. (2021) [14]) and the alcoholism model with two infected compartments, Mu'tamar Khozin(2018) [8] are presented.

Smoker model

$Pv(t)$ represents the population of potential smokers, $Ov(t)$ denotes the population of occasional smokers, $Sv(t)$ indicates the population of active smokers, $Qt(t)$ refers to the population of individuals who have temporarily quit smoking, and $Qp(t)$ signifies the population of individuals who have quit smoking permanently.

The model equations are

$$\frac{d(Pv)}{dt} = \Lambda - \left(\beta(PvSv)\right) - \mu Pv - \left(u1 + u4\right)Pv$$
$$\frac{d(Ov)}{dt} = \left(\beta(PvSv)\right) - (\alpha_1 + \mu)Ov - \left(u2 + u4\right)Ov$$
$$\frac{d(Sv)}{dt} = (\alpha_1)Ov + (\alpha_2)SvQt - (\gamma + \mu)Sv - \left(u3 + u4\right)Sv \quad (1)$$
$$\frac{d(Qt)}{dt} = -\left(\alpha_2 SvQt\right) - \left(\mu Qt\right) - \left(u2 + u4\right)Qt + \left(\gamma\left(1 - \sigma\right)Sv\right)$$
$$\frac{d(Qp)}{dt} = \left(\gamma\left(\sigma\right)Sv\right) - \left(\mu Qp\right) + \left(\left(u1 + u4\right)Pv\right) + \left(\left(u2 + u4\right)Ov\right) + \left(\left(u3 + u4\right)Sv\right) + \left(\left(u2 + u4\right)Qt\right)$$

The base values of the parameters are

$$\Lambda = 1; \mu = 0.001; \beta = 0.14; \alpha_1 = 0.002; \alpha_2 = 0.0025; \gamma = 0.8; \sigma = 0.1;$$
$$u1 = 0.6; u2 = 0.0; u3 = 0.65; u4 = 0;$$

u2 is chosen as the bifurcation parameter for the bifurcation analysis. u1 u2 u3 and u4 are chosen as the control variables.

Alcoholics Model

The model equations are

$$\frac{ds}{dt} = \mu\left(1 - s\right) - \beta\left(1 - u\right)s\left(a1 + a2\right)$$
$$\frac{d(a1)}{dt} = \beta s\left(1 - u\right)a1 - (\mu + k + \xi)a1$$
$$\frac{d(a2)}{dt} = \beta s\left(1 - u\right)a2 - (\mu + \xi)a2 + ka1 \quad (2)$$
$$\frac{dr}{dt} = \xi(a1 + a2) - \mu r$$

Here s, a1, a2, and r represent the scaled values of susceptible individuals, admitted infected,
non-admitted infected and recovered individuals.

The base parameter values are $\mu = 0.2; \beta = 0.5; k = 0.051; \xi = 0.1; \phi = 100; u = 0.5.$

2.Bifurcation analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[15]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[16]). This program detects Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha) \quad (3)$$

$x \in R^n$ Let the bifurcation parameter be α Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \quad (4)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \quad (5)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector w_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x, \alpha) @ I_n) = 0 \quad (6)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[17]; 2009[18]) and Govaerts [2000] [19]

3.Multiobjective Nonlinear Model Predictive Control (MNLMPC)

Flores Tlacuahuaz et al (2012) [20] developed a multiobjective nonlinear model predictive control (MNLMPC) method that is rigorous and does not involve weighting functions or additional constraints. This procedure is used for performing the MNLMPC calculations Here $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) represents the variables that need to be minimized/maximized simultaneously for a problem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \qquad (7)$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. This MNLMPC procedure first solves the single objective optimal control problem independently optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ individually. The minimization/maximization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then the optimization problem that will be solved is

$$\min(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2 \qquad (8)$$

$$subject\ to\ \frac{dx}{dt} = F(x,u);$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017) [21] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006) [22]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005) [23].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^* at various time intervals t_i. The subscript *i* is the index for each time step.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$ and get the control values for various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j.

Sridhar (2024) [24] proved that the MNLMPC calculations to converge to the Utopia solution when the bifurcation analysis revealed the presence of limit and branch points . This was done by imposing the singularity condition on the co-state equation (Upreti, 2013) [25]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1 - q_1^*)^2 + (q_2 - q_2^*)^2$.

The multiobjective optimal control problem is

$$\min \quad (q_1 - q_1^*)^2 + (q_2 - q_2^*)^2 \quad subject\ to\ \frac{dx}{dt} = F(x,u) \qquad (9)$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 2(q_1 - q_1^*)\frac{d}{dx_i}(q_1 - q_1^*) + 2(q_2 - q_2^*)\frac{d}{dx_i}(q_2 - q_2^*) \qquad (10)$$

The Utopia point requires that both $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 0 \qquad (11)$$

the optimal control co-state equation (Upreti; 2013) [25] is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) - f_x\lambda_i; \quad \lambda_i(t_f) = 0 \qquad (12)$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i) = -f_x\lambda_i; \lambda_i(t_f) = 0 \qquad (13)$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This, coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the only solution is the Utopia solution.

4.Results and Discussion

Bifurcation analysis for the Smoker model revealed the existence of a limit point at

(*Pv, Ov, Sv, Qt, Qp,u2*) values of (1.663894, 0, 0, 5.834, 992.5019, -0.001). This is shown in Fig. 1. Here u2 is chosen as the bifurcation variable.

For the alcoholics model the bifurcation analysis revealed the existence of a branch point at

(*s,a1,ar,r,u*) values of (1.0,0.0, 0.0, 0.0, 0.4). This is shown in Fig. 2. u is chosen as the bifurcation parameter.

For the MNLMPC calculation, in the smoker model, $\sum_{t_{i=0}}^{t_i=t_f} Qp(t_i)$ was maximized and $\sum_{t_{i=0}}^{t_i=t_f} Sv(t_i)$ was minimized individually, and each led to values of 2000 and 0. The multiobjective optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} Qp(t_i) - 2000)^2 + (\sum_{t_{i=0}}^{t_i=t_f} Sv(t_i) - 0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for u1 u2 u3 and u4 were 0.2318, 0.01082, 0.4571, and 0.010441.

The various profiles for this MNLMPC calculation are shown in Figs. 3a,3b,3c and 3d. The obtained control profile of u1 u2 u3 and u4 exhibited noise (Fig. 3e and 3f.). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of the profiles are shown in Fig. 3g and 3h. The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024) [24], which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

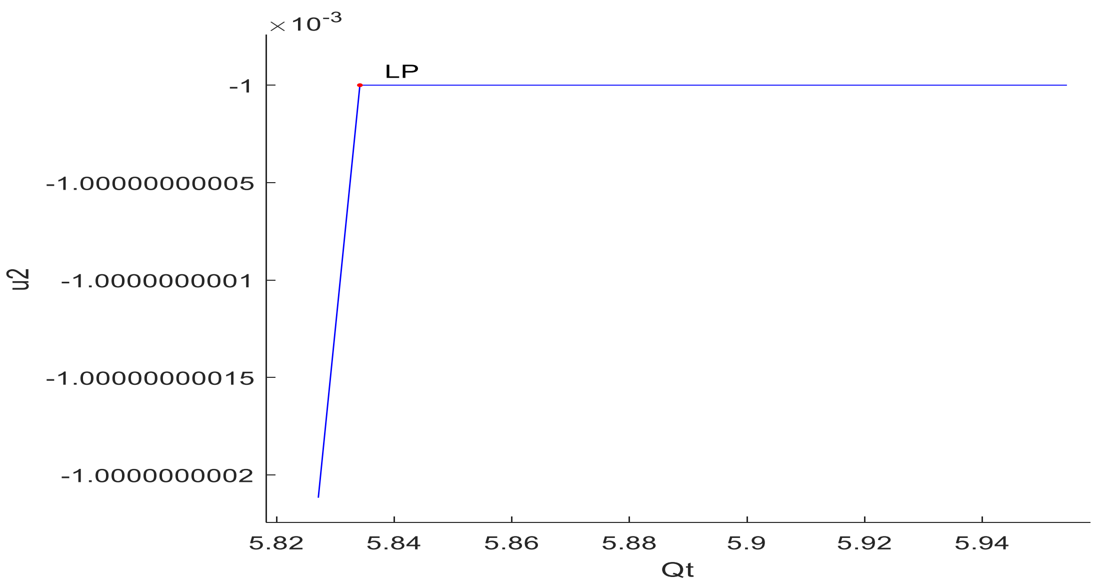

Fig. 1 Bifurcation Diagram for the Smoker Model

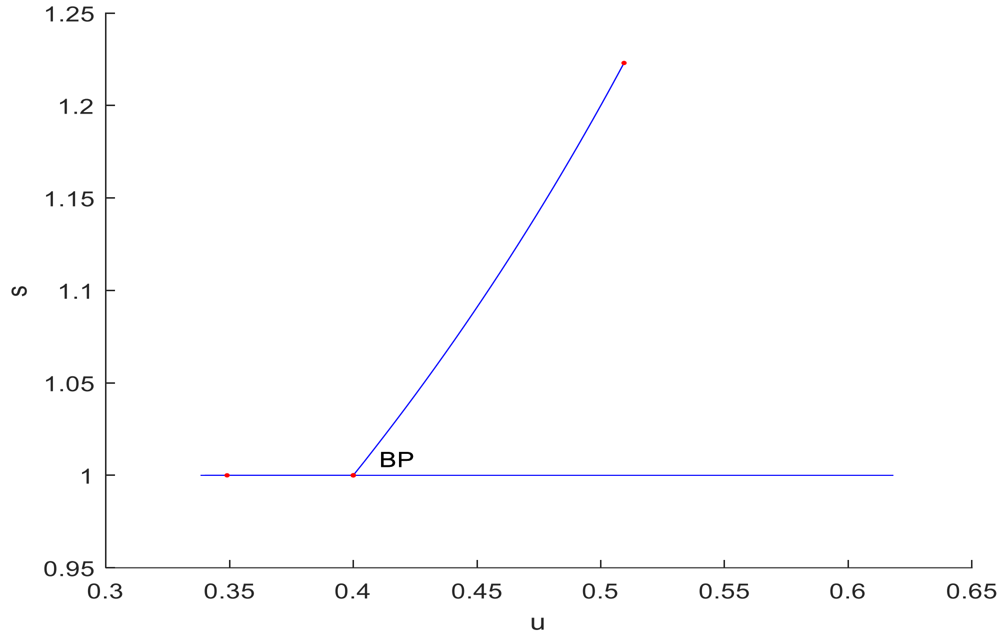

Fig. 2 Bifurcation Diagram for the Alcoholics Model

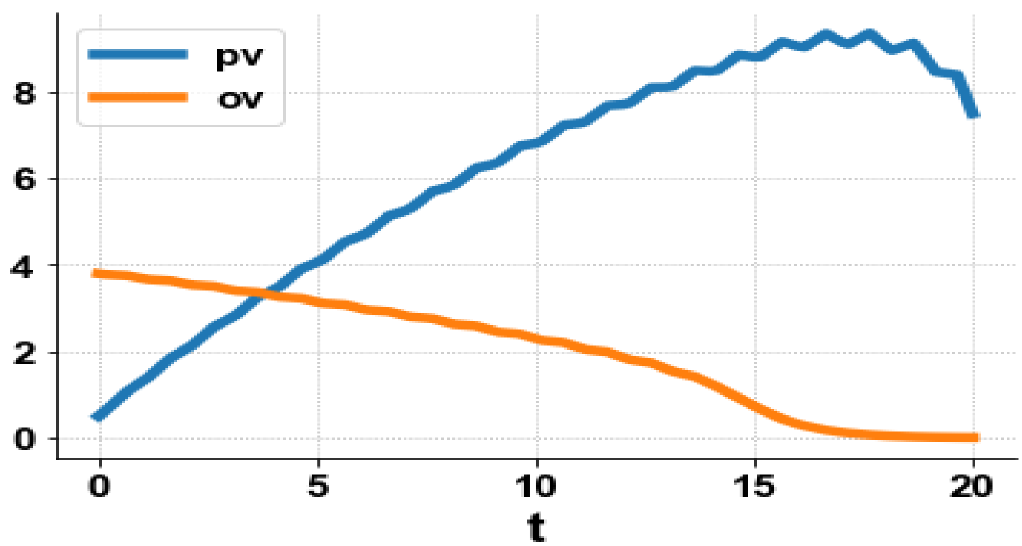

Fig. 3a MNLMPC smoker model Pv, Ov vs t

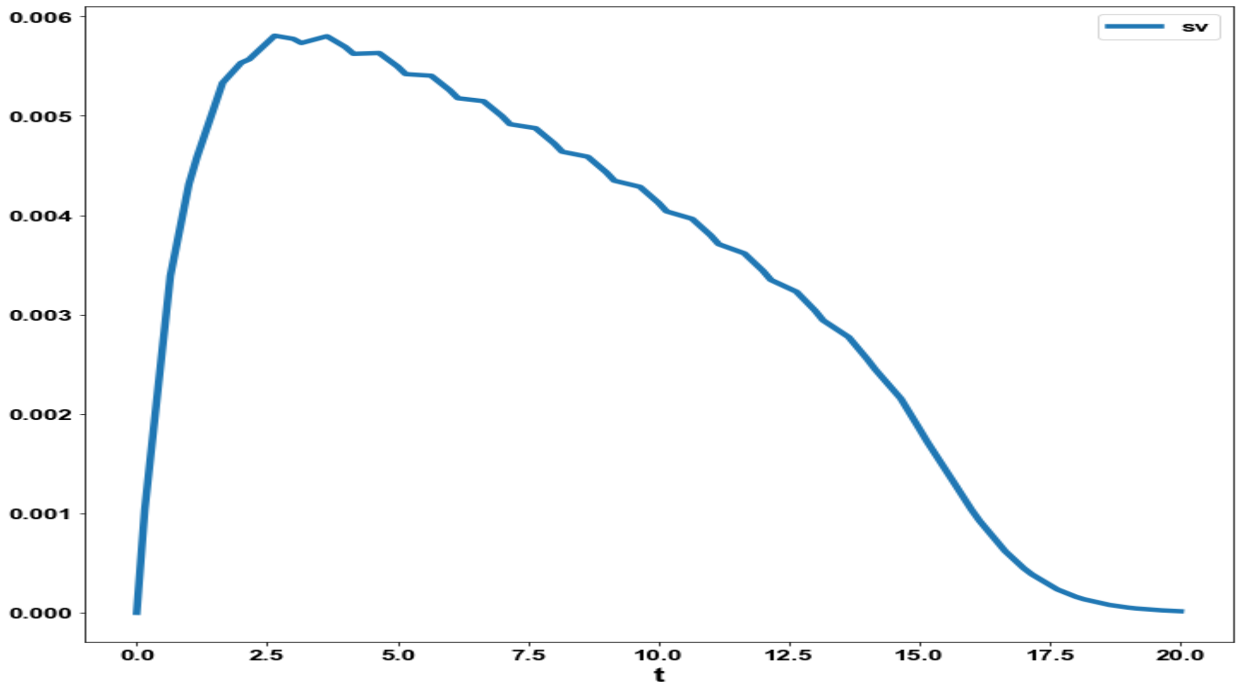

Fig. 3b MNLMPC smoker model Sv vs t

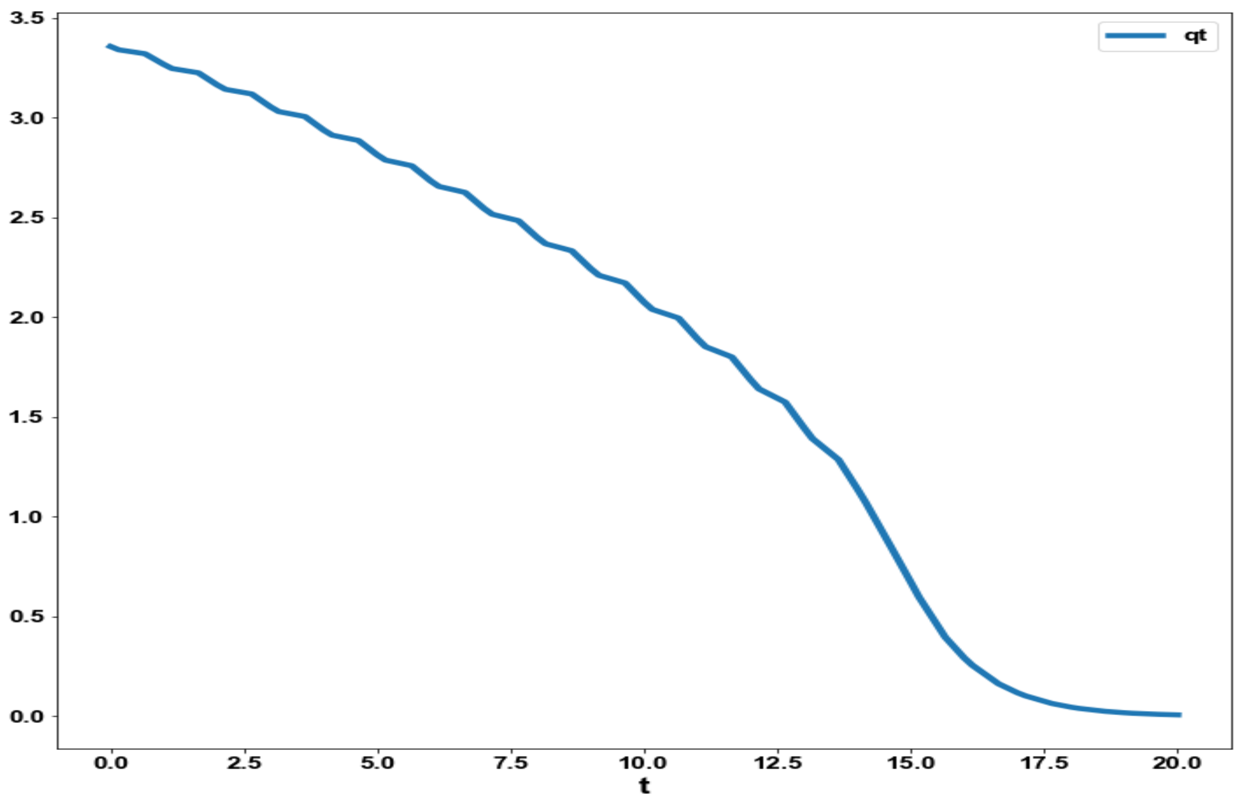

Fig. 3c MNLMPC smoker model Qt vs t

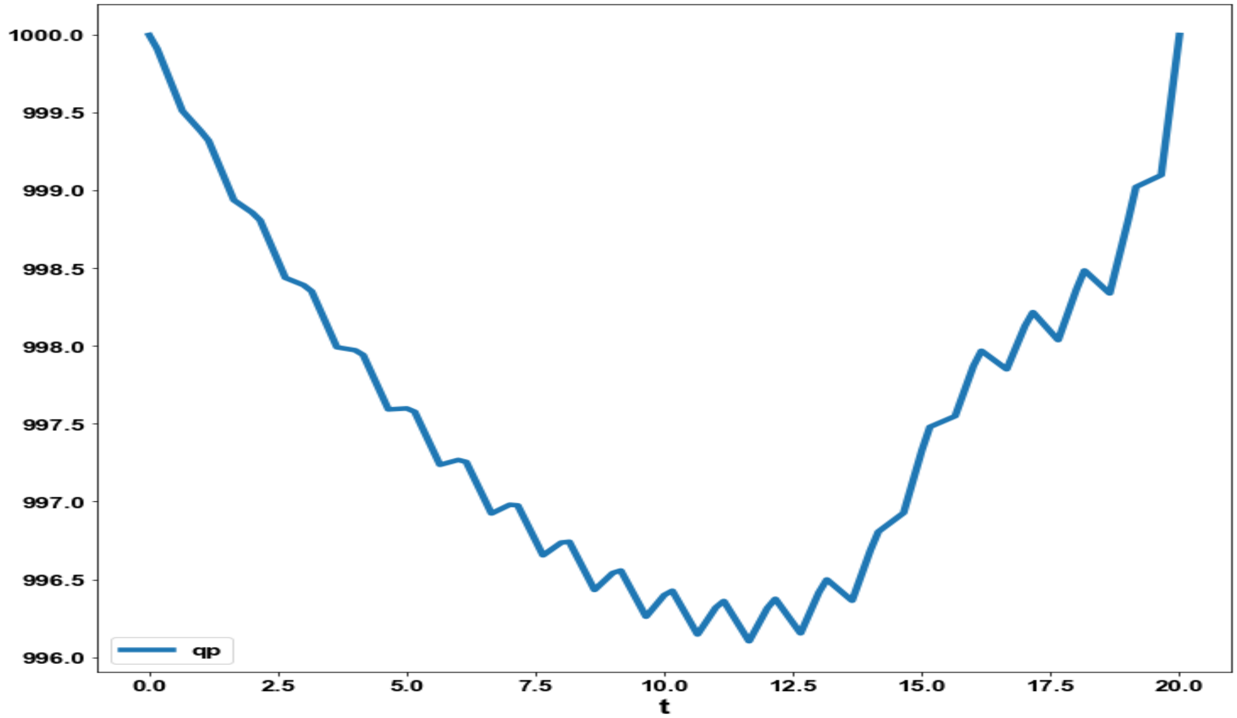

Fig. 3d MNLMPC smoker model Qp vs t

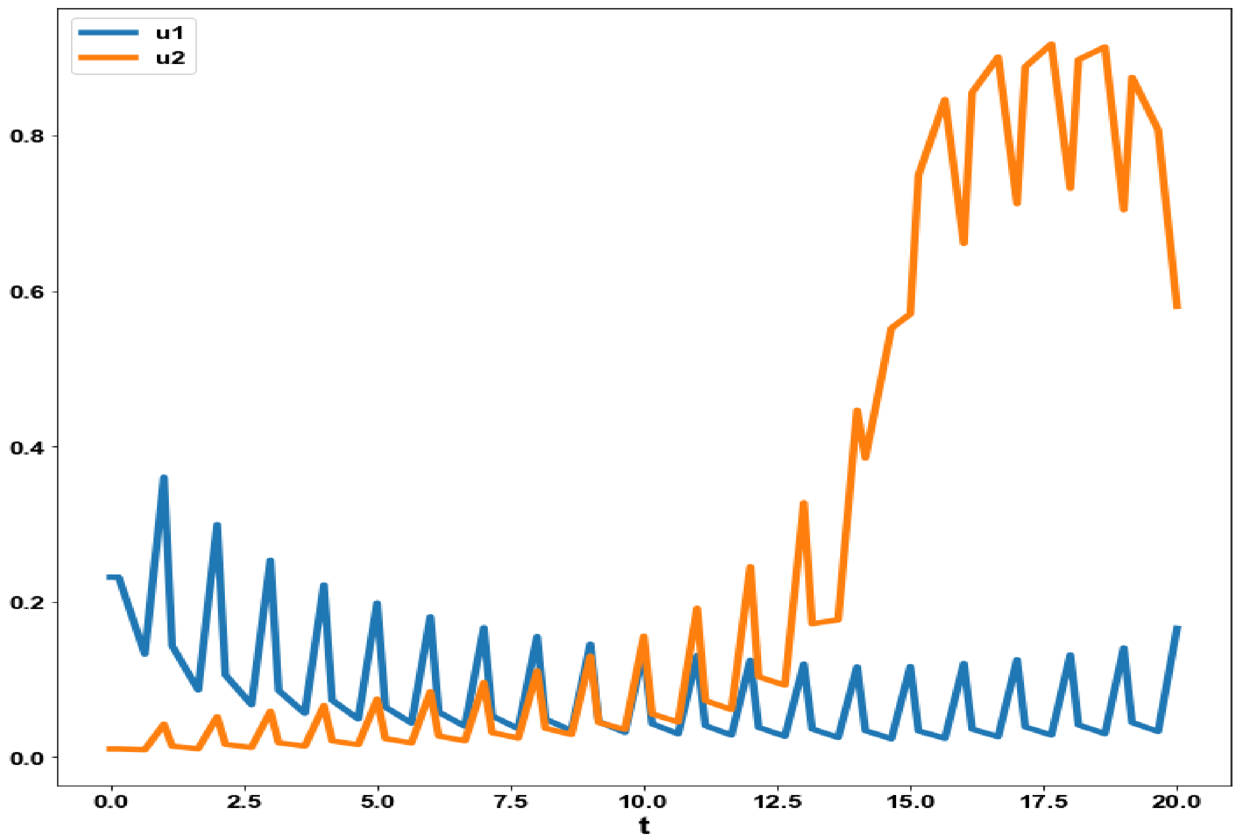

Fig. 3e MNLMPC smoker model u1 u2 (noise exhibited)

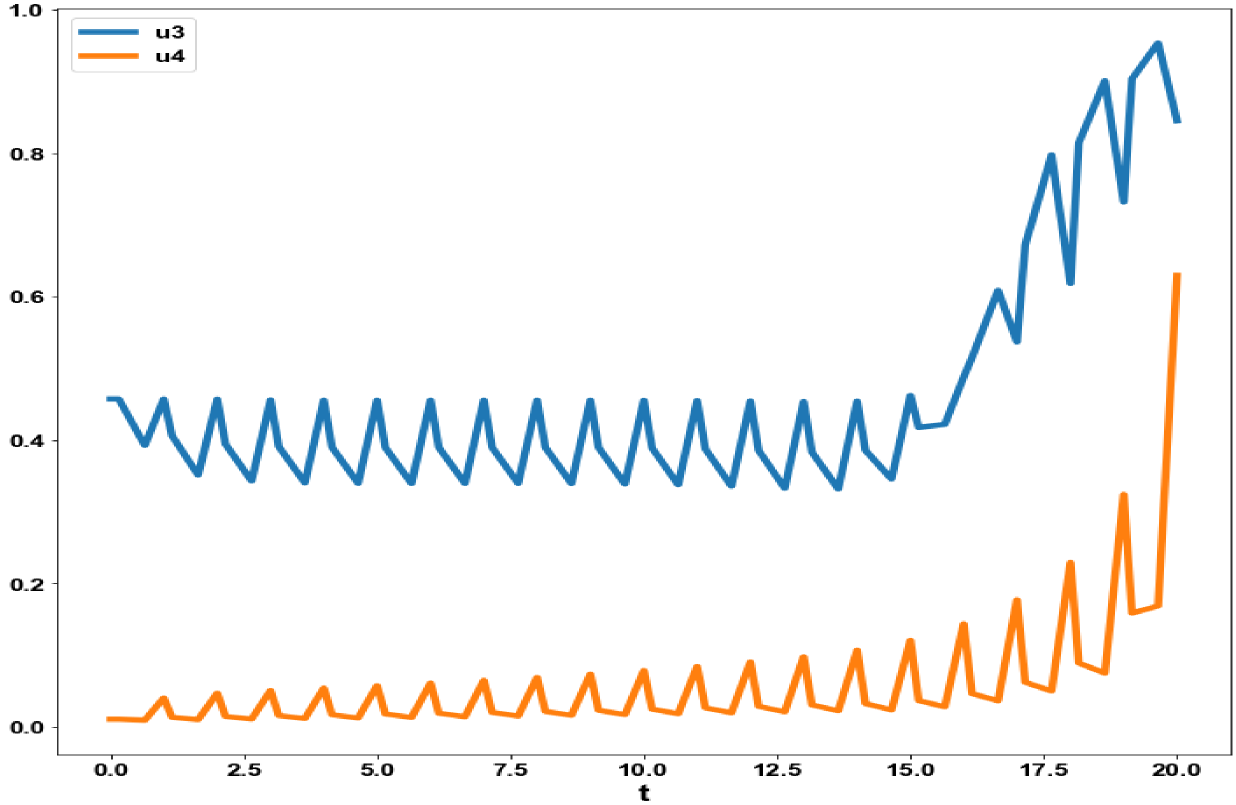

Fig. 3f MNLMPC smoker model u3 u4 (noise exhibited)

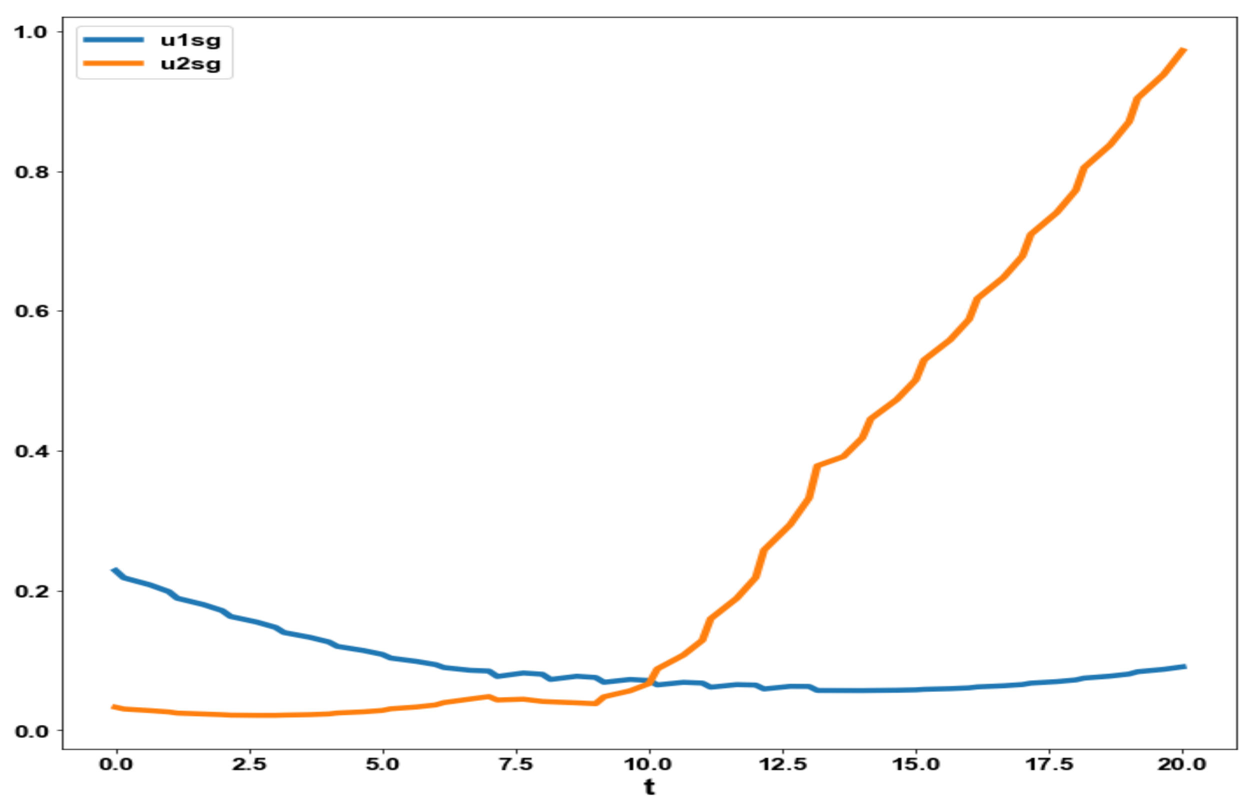

Fig. 3g MNLMPC smoker model u1 u2 (with Sazitzky Golay filter) vs t (noise eliminated)

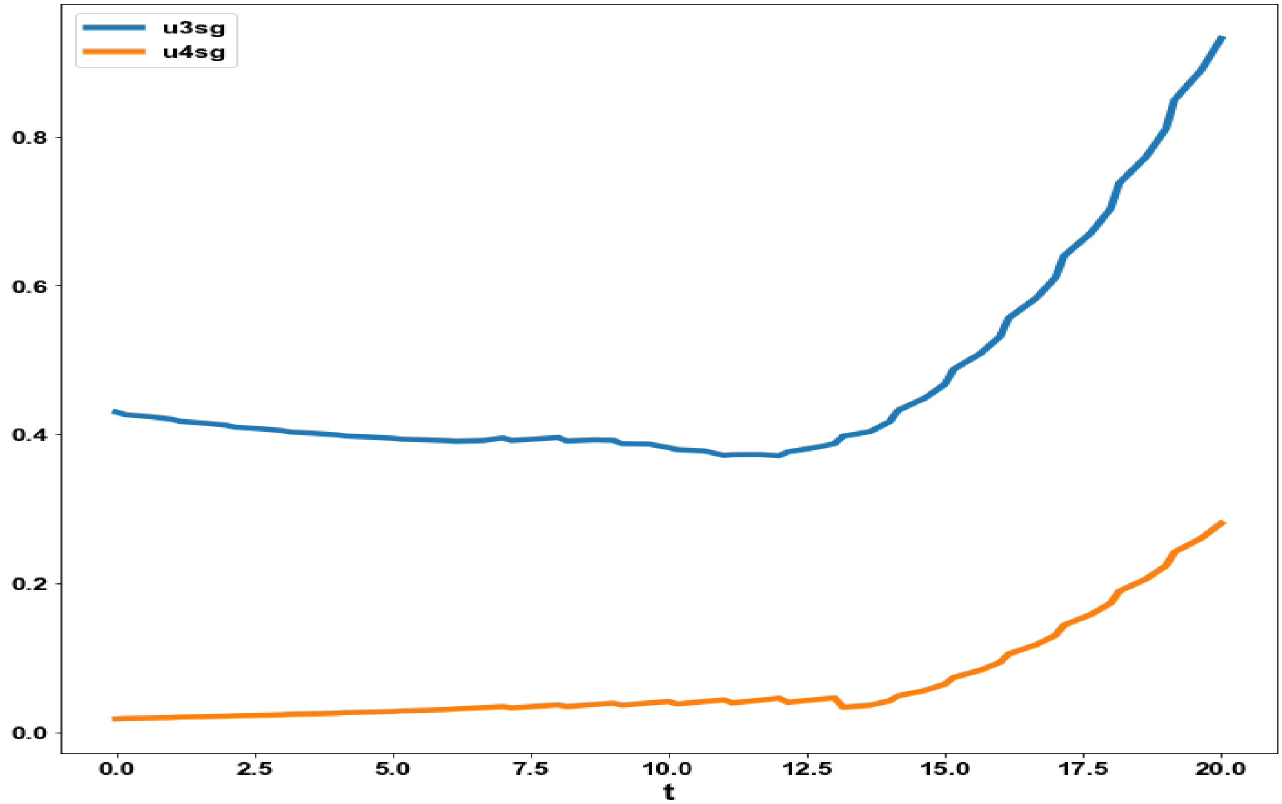

Fig. 3h MNLMPC smoker model u3 u4 (with Sazitzky Golay filter) vs t (noise eliminated)

For the MNLMPC calculation, in the alcoholics model, $\sum_{t_{i=0}}^{t_i=t_f} a1(t_i), \sum_{t_{i=0}}^{t_i=t_f} a2(t_i)$ were minimized individually, and each led to a value of 0. The multiobjective optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} a1(t_i)-0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} a2(t_i)-0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for u was 0.8801.

The various profiles for this MNLMPC calculation are shown in Figs. 4a 4b. The obtained control profile of u vs t exhibited noise (Fig. 4c). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of the profile is shown in Fig. 4d. The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024) [25], which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

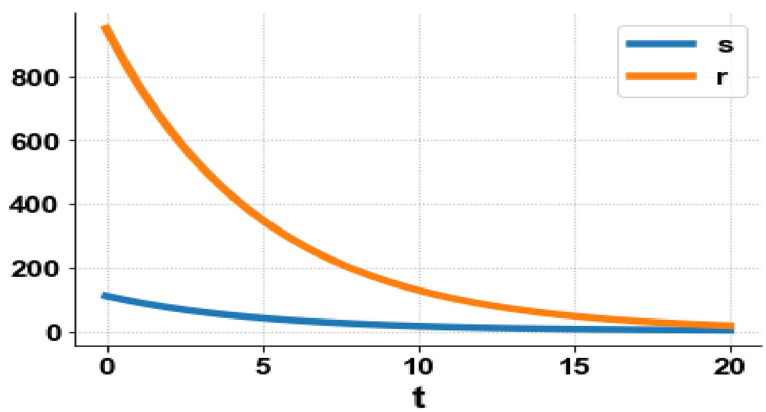

Fig. 4a MNLMPC alcoholics model s, r vs t

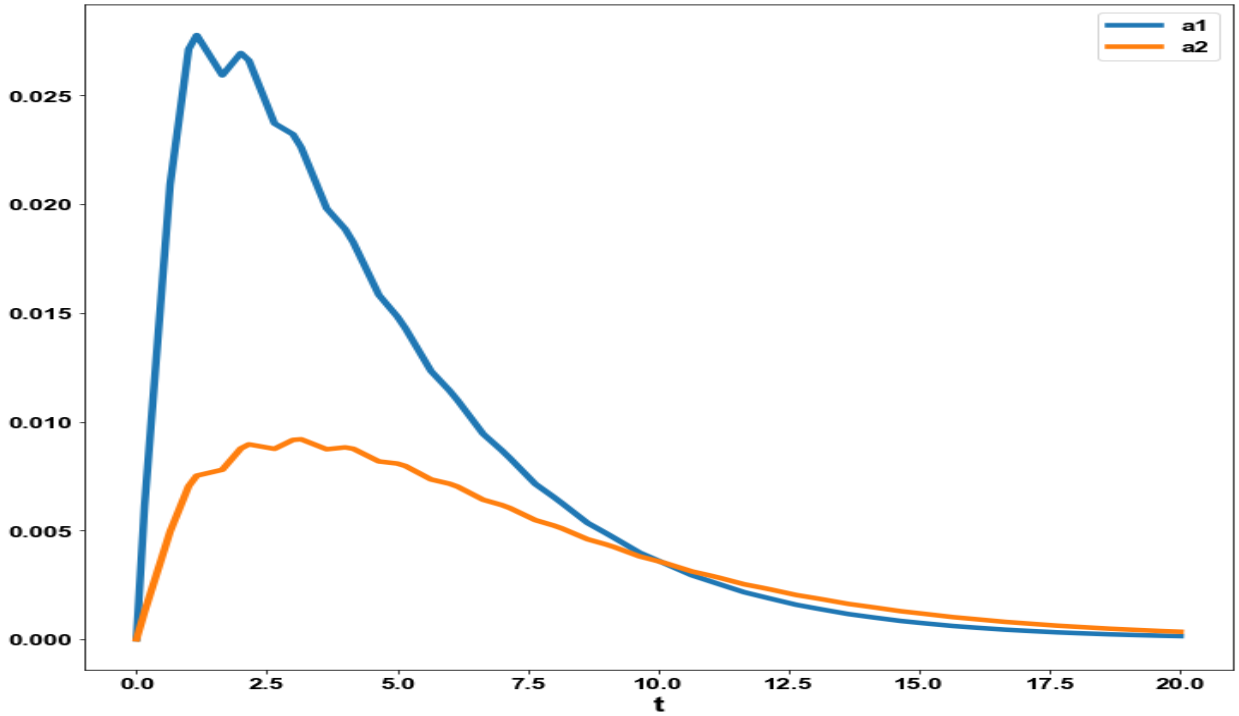

Fig. 4b MNLMPC alcoholics model a1, a2 vs t

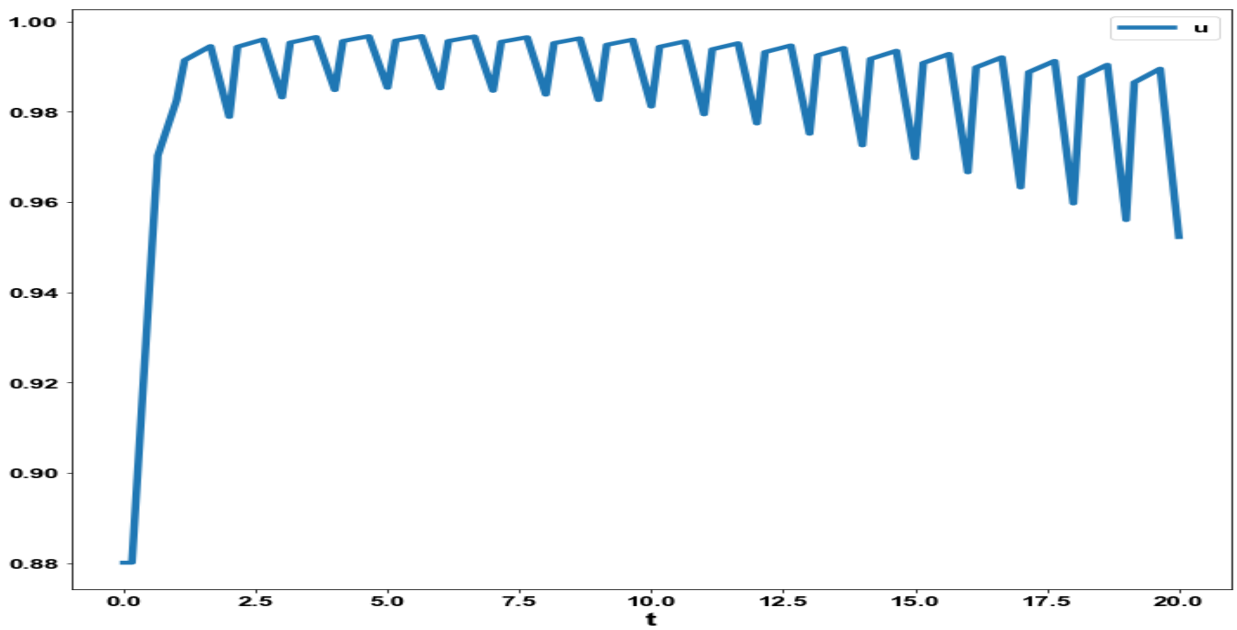

Fig. 4c MNLMPC alcoholics model u vs t (noise exhibited)

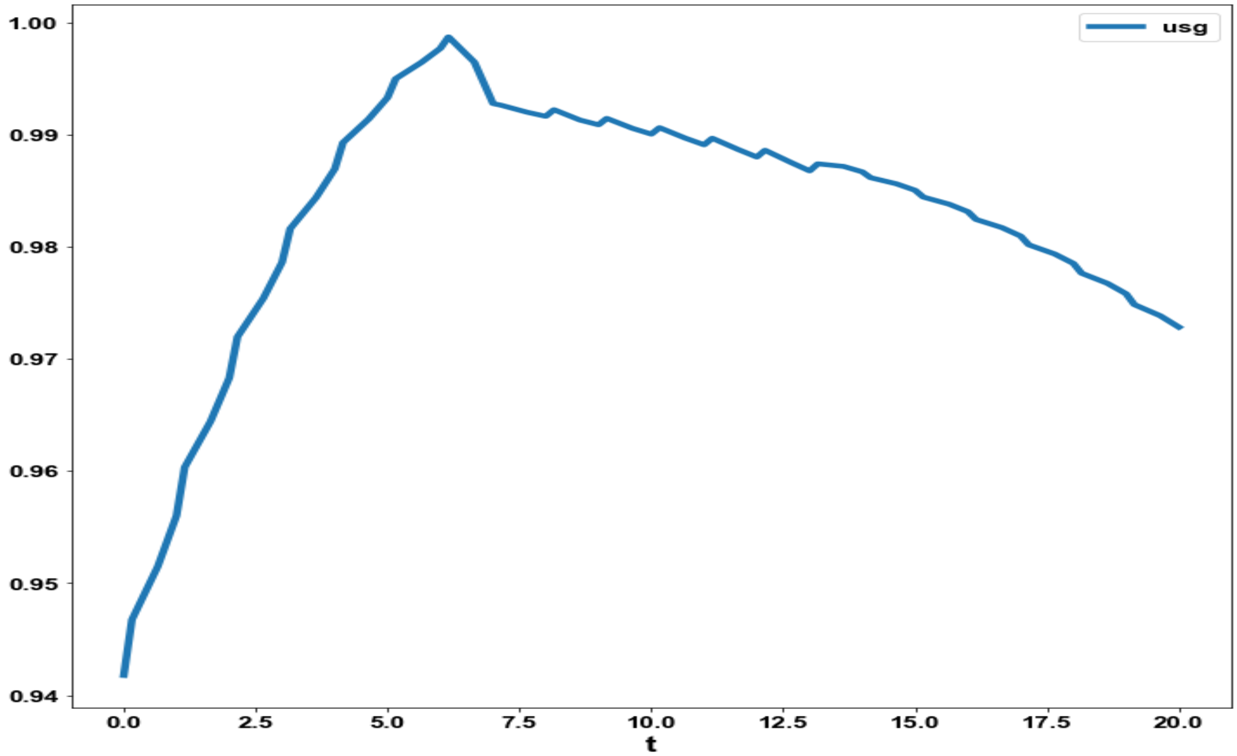

Fig. 4d u vs t (with Sazitzky Golay filter) vs t (noise eliminated)

4. Conclusions

Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on a cigarette smoking and alchholics models. The bifurcation analysis revealed the existence of a limit and a branch point. The limit and branch points (which causes multiple steady-state solutions from a singular point) is very beneficial because it enables the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control(MNLMPC) for a dynamic models involving cigarette smoking and alcoholism is the main contribution of this paper.

Data Availability Statement

All data used is presented in the paper

Conflicts of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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